Assignment n.3
Due April 10th

1) Consider a political economy model where the possible policies are ordered on the line from -1 to 1: All citizens have preferences lying somewhere on the line. All citizens must vote and they vote for the candidate whose platform lies closest to their preference point. Suppose that there are 3 candidates that must simultaneously select their platform. The candidate receiving the largest number of votes is the winner. Verify that there does not exist a Nash equilibrium.

2) Consider a Cournot duopoly in which the two firms simultaneously choose a quantity to produce: $q_1$ for firm 1 and $q_2$ for firm 2. The inverse demand function for this market is $P(Q) = 48 - \frac{1}{2}Q$; where $Q$ is the total quantity in the market: $Q = q_1 + q_2$. The marginal cost of production is $c_1 = 6$ for firm 1 and $c_2 = 6$ for firm 2. Solve for the Nash equilibrium of this game (the quantity choices of both firms).

Suppose now that the two firms move sequentially: first, firm 1 makes its choice of production, next firm 2, observed firm's 1 action, decides its own production level. What are firm 2's strategies? What level of production would maximise firm 2's profit if firm 2 played alone? Denote this level the monopoly level $q_2^M$. Show that there exists a Nash Equilibrium where firm 1's production is zero and firm 2 produces the monopoly level. Is such equilibrium reasonable?

3) Consider the Bertrand model of duopoly with transportation costs. Two firms compete in prices by simultaneously selecting prices $p_1$ and $p_2$: Firm 1 is located at mile 0 and firm 2 is located at mile 1. Consumers are uniformly distributed between mile 0 and mile 1 and incur a cost of 1 for each mile traveled to reach the selected firm. Consumers purchase from the firm with the lowest total cost (price plus travel cost) and have a demand equal to 1. The firms have marginal costs of production given by $c_1$ and $c_2$. Firms cannot run negative profits: in this case they prefer to shut down. Initially assume that $|c_1 - c_2| \leqslant 3$: what are the equilibrium price choices $p_1$ and $p_2$? What happens if $|c_1 - c_2| > 3$; say $c_1 = 8$ and $c_2 = 4$? Specifically, what are the equilibrium price choices?